Nonlinearity of Regge trajectories in the scattering region

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Abstract

The nonlinearity of Regge trajectories at real negative values of the argument is discussed as their general QCD-inspired property. The processes of elastic diffractive scattering $p+p \to p+p$ and $\bar{p}+p \to \bar{p}+p$ at collision energies $\sqrt{s} > 50\,GeV$ and transferred momenta $0.01\,GeV^2 < -t < 3\,GeV^2$ are considered in the framework of the Regge-eikonal model [3]. By comparison of phenomenological estimates with available experimental data on angular distributions it is demonstrated that in this kinematical range the data can be satisfactorily described as if taking into account only two nonlinear C-even Regge trajectories with vacuum quantum numbers (f_2-reggeon and "soft" pomeron ones). It is also shown that their nonlinearity is essential and not to be ignored. The correspondence of the Kwiecinski $q\bar{q}$-pole [24] to the f_2-reggeon trajectory and the relevance of the Kirschner-Lipatov "hard" pomeron pole [31] to elastic diffraction is discussed.

Introduction

Our goal is to substantiate that nonlinearity of leading Regge trajectories at negative values of the argument (nonlinearity of trajectories in the resonance energy region is discussed in [1], [2]) following from some general requirements is essential and not to be neglected under considering strong interaction phenomena in the framework of Regge-based models. We will demonstrate this on the example of high energy elastic (anti)pp-scattering and will exploit for this purpose the Regge-eikonal model [3]. We do not intend to compete with [4], [5], [6], [7], [8], [9], [10] and many other authors in getting the lowest value of χ^2/d.o.f. over all available data on angular distributions at high energies.

The nonlinearity of the leading trajectory was demonstrated in the experiment on measurement of single diffraction cross-sections [11]. Also, the use of the effective nonlinear dipole pomeron trajectory provided the successful description of existing data on high-energy elastic (anti)pp-scattering [5], on photoproduction of vector mesons [12], [13] and deeply virtual Compton scattering [14].

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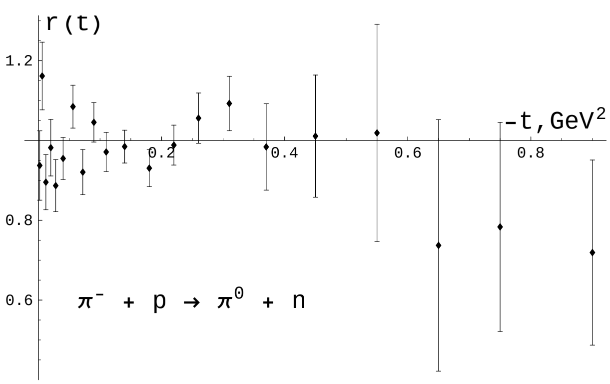
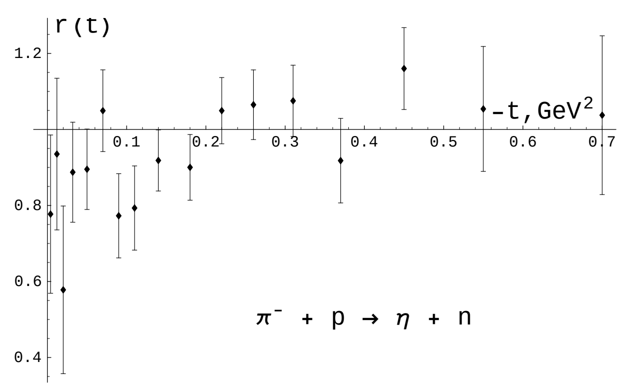
Note also that authors [4], [15] as well as many others insist on the linearity of some leading Regge trajectories in the Euclidean domain at least at $-3\,GeV^2 < t < 0$. The first phenomenological argument is a natural desire to continue Chew-Frautschi plots to the region of negative values of the argument. But all poles corresponding to resonances are situated on different unphysical sheets since they possess not only the mass but also the nonzero width. Hence, appropriate points on any Chew-Frautschi plot pertain to different branches of the corresponding analytic function and straight continuation to the region below the lowest threshold is not correct from the analytical point of view (although it can be used, for example, for rough estimation of the intercept value). We have to conclude that hadron spectroscopy does not provide absolutely reliable grounds for determining the behavior of Regge trajectories in the Euclidean domain. The second "evidence" of the linearity originates from the data on exchange processes $\pi^- + p \to \pi^0 + n$ and $\pi^- + p \to \eta + n$ in the Born approximation [16]. Since within this approach only two of the leading reggeons (ρ and a_2) give contribution to the amplitude and we can consider these trajectories approximately equal (as a consequence of the weak degeneracy following from the hadron spectroscopy data) one could try to extract them directly from the corresponding high-energy angular distributions [17]. The appropriate formula is

$$\alpha^\rho(t) \equiv \alpha^\rho_{ij}(t) = 1 + \frac{\ln\left(\frac{d\sigma}{dt}^i \Big/ \frac{d\sigma}{dt}^j\right)}{2\ln\frac{s_i}{s_j}}\,, \tag{1}$$

where $\frac{d\sigma}{dt}^i$ is the differential cross-section at the c.m.s. collision energy $\sqrt{s_i}$. For any different s_i and s_j function $\alpha^\rho(t)$ must be the same (if the Born approximation works) and, consequently, the function $r(t) \equiv (1-\alpha^\rho_{23}(t))/(1-\alpha^\rho_{12}(t))$ (here $s_1 = 40.78\,GeV^2$, $s_2 = 122.57\,GeV^2$ and $s_3 = 375.65\,GeV^2$) must be strictly equal to unity for both the above-mentioned processes at any argument value. However, this is not evident after extraction this function from the data on reactions $\pi^- + p \to \pi^0 + n$ and $\pi^- + p \to \eta + n$ (fig. below). The only conclusion





which an unprejudiced person can draw looking at these pictures is that amount and quality of the available data does not allow to verify the validity of the Born approximation for an unambiguosly good description of the exchange processes in the considered region and, as a consequence, the linearity of the ρ/a_2-trajectory is not guaranteed. Also note that the best fit to the trajectory extracted from the earlier data on $\pi^- + p \to \eta + n$ [18] under exploiting the Born approximation was obtained for nonlinear parametrization [19], [20]

$$\alpha(t) = \alpha(\infty) + \frac{[\alpha(0)-\alpha(\infty)]^2}{\alpha(0)-\alpha(\infty)-t\alpha'(0)} \tag{2}$$

with $\alpha(\infty) = 0$.

Our viewpoint is based on the conviction that QCD is the fundamental theory of strong interaction. This rather general requirement imposes restrictions on the behavior of Regge trajectories in the range of the perturbative QCD validity since at large transferred momenta exchanges by single reggeons must turn into exchanges by colour-singlet parton combinations. If we assume that the reggeon exchange giving the leading contribution to the eikonal at high energies (pomeron) turns into some multi-gluon exchange in the perturbative range we will come to [21], [22], [23], [31] $\lim_{t\to-\infty}\alpha_P(t) = 1$. In the case of the quark-antiquark pair (f_2-reggeon, a_2-reggeon etc.) one obtains [24] $\lim_{t\to-\infty}\alpha_f(t) = 0$.[1]

This property of Regge poles (tending to constant at $t \to -\infty$) is quite general and follows from the fact of their invariance relative to the renormalization group transformations (the requirement of renorm-invariance is well-grounded by observability of bound states and resonances). The general solution of the renorm-group differential equation for the renorm-invariant quantity (in the case of massless fields) $f(\frac{t}{\mu^2}\,,\ \alpha_s(\mu^2))$ is of the form [26]

$$f\left(\frac{t}{\mu^2}\,,\ \alpha_s(\mu^2)\right) = \Phi\left(\frac{t}{\mu^2}e^{K(\alpha_s(\mu^2))}\right) \tag{3}$$

where $\Phi(x)$ is an arbitrary function analytic in the region defined by the analyticity in t, μ is the renormalization scale, $\alpha_s(\mu^2)$ is the effective coupling constant, $K'(\alpha_s(\mu^2)) = (\mu^2\frac{\partial\alpha_s(\mu^2)}{\partial\mu^2})^{-1}$. For any quantum field model with asymptotic freedom (3) leads in the perturbative sector ($\mu^2 = -t$, $t \to -\infty$) to

$$\lim_{t\to-\infty} f(t) = const\,. \tag{4}$$

Actually, this corresponds to the existence of the free field limit as $\alpha_s(t) \to 0$. In the theory of potential scattering the squared effective radius of interaction corresponding to the reggeon exchange [19], [27] $R^2 \sim \alpha'(t)(2\alpha(t) + 1) \to 0$ at $t \to -\infty$, $\alpha(t) \to const$. This purely quantum mechanical result is in agreement with the property of asymptotic freedom at short distances (in the case of linear trajectory $\alpha(t)$ ($\alpha'(t) > 0$) $R^2 \to -\infty$ at $t \to -\infty$).

As a consequence of (4) the essential nonlinearity of the Regge trajectories takes place in the range $-\infty < t < 0$. This nonlinearity is their fundamental property. Besides, we assume the monotony of the leading trajectories at negative values of the argument. Such a situation takes place, for example, if the dispersion relations do not need more than one subtraction and if the imaginary part of the corresponding function is non-negative [28].

It is shown in [29] that the assumption of the linearity of Regge trajectories leads to the situation when the diffractive pattern at high energy (anti)proton-proton elastic scattering can be described in the framework of the Regge-eikonal approach only after introduction of several vacuum reggeons giving contributions to the eikonal (one needs not less than three C-even poles with intercepts higher than unity). The use of nonlinear trajectories allows us to curtail the number of reggeons essential for an acceptable description of the data on high energy elastic nucleon-nucleon scattering. Namely, using only two trajectories with vacuum quantum numbers

[1] The universality of the asymptotic vanishing at $t = -\infty$ of meson trajectories seems to contradict the existence of (pseudo)scalars. In fact, if $\alpha_\pi(t)$ behaved like $\alpha_f(t)$ at $t \to -\infty$ it could not be monotonic function of t. Possible way to preserve the monotony is to suppose that at $t \to -\infty$ the trajectory tends to some negative value. Similar behavior takes place, e.g., in the phenomenological studies of the trajectories containing heavy quarkonia [25]. To what extent is it possible in QCD remains unclear.

and intercepts higher ("soft" pomeron) and lower (f_2-reggeon) than unity we will be able to describe available experimental data (including the diffractive pattern) in a wide kinematical range $\sqrt{s} > 50\,GeV$, $0.01\,GeV^2 < -t < 3\,GeV^2$ (at $-t < 0.01\,GeV^2$ the Coulomb interference takes place, at $\sqrt{s} < 50\,GeV$ or $-t > 3\,GeV^2$ we can not ignore the contributions from other poles). Of course, taking into account only C-even reggeons we can not discern pp- and $\bar{p}p$-scattering. But at $\sqrt{s} > 50\,GeV$ the difference in cross-sections for these reactions is much more fine effect than the elastic scattering phenomenon itself and we can neglect it. The very fact that we have managed to satisfactorily reproduce experimental data in the framework of such a simple scheme is quite encouraging and points to the agreement between general QCD-related theoretical conclusions and experiment.

The QCD pomeron

Regge trajectories which are tightly related to the hadron spectroscopy deal, generally, with confinement of quarks and gluons. Thus, without a great progress in the solution of this outstanding problem the QCD theory of Regge trajectories is still in its primordial stage.

It is natural that in the absence of a regular non-perturbative technique some progress is limited by perturbative calculations. In the literature the wide attention is being paid to so-called "BFKL pomeron" (or other "BFKL reggeons") [30]. In this approach one strives to formulate some kind of Bethe-Salpeter equation for the gluon-gluon scattering amplitude (Green function) in which "t-channel" gluons are actually gluon Regge trajectories ("reggeized gluons") that have to be preliminary calculated from another Bethe-Salpeter equation for the colour-octet t-channel. This trick can make an impression that, say, the pomeron trajectory is not just a 2-gluon exchange (with account of interaction between these two gluons via "ladder rung exchanges") but contains many-gluon configurations in the t-channel as well. However, the very "reggeized gluons" are related to no more than two gluons each. That is one deals with no more than 4 usual gluons in the t-channel.

The use of fixed number of gluon exchanges seems to be justified at $|t|$ large enough, where the elementary short-distance structure of reggeons has to show up. As to small values of t it is clear that due to genuinely strong interaction one cannot limit the problem by any fixed number of exchanged partons. Certainly, one can argue about "valence" gluons but these have to be essentially non-perturbative and different from the "slightly reggeized" gluons mentioned above. It is even quite probable that the corpuscular language in this quasi-classical region seases to be adequate and undulatory gluonic fields are more relevant.

In spite of such a little bit gloomy landscape one still can try to account (when describing the scattering data in terms of Regge exchanges) for such a distinctive QCD prediction as asymptotic constancy of Regge trajectories in deeply Euclidean region, $-t \to \infty$ (in other words, the nonlinearity of the pomeron and secondary reggeon trajectories which was already mentioned in the previous section). Via solving the Bethe-Salpeter-like equation for the gluon-gluon scattering amplitude it was obtained in [31] that

$$\alpha_P^{hard}(t) = 1 + \frac{12\,\ln 2}{\pi}\alpha_s(t)\left[1 - \alpha_s^{2/3}(t)\left(\frac{7\zeta(3)}{2\,\ln 2}\right)^{1/3}\left(\frac{3/4}{11 - 2/3\,n_f}\right)^{2/3}\right]\,, \quad t \to -\infty\,. \qquad (5)$$

The second term in brackets is approximately equal to 0.09 at $t = -M_Z^2$ when $\alpha_s(-M_Z^2) \approx$

0.118 (here $M_Z \approx 91.2\,GeV$ is the Z-boson mass and $n_f = 5$) and such an expansion is justified at this scale (at $t = -6\,GeV^2$, $\alpha_s(-6\,GeV^2) \approx 0.3$ this term is about 0.17). Even at $t = -M_Z^2$ the value of $\alpha_P^{hard}(-M_Z^2) \approx 1.28$ is quite high ($\alpha_P^{hard}(-6\,GeV^2) \approx 1.66$). If one assumes the monotony of $\alpha_P^{hard}(t)$ the intercept, $\alpha_P^{hard}(0)$, has to lie even higher. The last statement agrees with a rough estimate of the lower bound for the BFKL pomeron intercept value $\alpha_{min}(0) - 1 \approx 0.3$ obtained in [32]. Such a value can be hardly relevant for the data on elastic diffraction.

Some time ago a NLO result was obtained for the pomeron intercept [33]

$$\alpha_P(0) = 1 + \frac{12\,\ln 2}{\pi}\alpha_s \left(1 - \frac{20}{\pi}\alpha_s\right) . \tag{6}$$

This expression cannot be accepted as a true value of the pomeron intercept as it depends on the renormalization scheme and arbitrary renormalizaton scale μ via α_s. As was argued recently [34] the intercept of any Regge trajectory has to be strictly independent of the QCD coupling constant.

These curcumstances enforce us (while waiting for further theoretical progress) to try some purely phenomenological ansatz for the pomeron trajectory which retains but one feature of (5): it tends to 1 at $|t|$ high enough.

The model

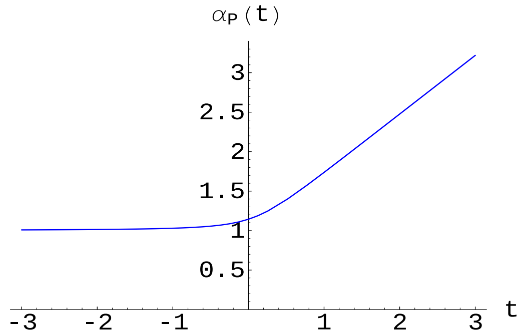
We surmise that at high energies ($\sqrt{s} > 50\,GeV$) it is enough to take into account only two Regge poles ("soft" pomeron and f_2-reggeon). Then the eikonal is of the form (below we have chosen $s_0 = 1\,GeV^2$)

$$\delta(s,t) = \left(i + tg\frac{\pi(\alpha_P(t)-1)}{2}\right)\beta_P(t)\left(\frac{s}{s_0}\right)^{\alpha_P(t)} + \left(i + tg\frac{\pi(\alpha_f(t)-1)}{2}\right)\beta_f(t)\left(\frac{s}{s_0}\right)^{\alpha_f(t)} . \tag{7}$$

Our phenomenological ansatz for the "soft" pomeron pole function is (see fig. below)

$$\alpha_P(t) = 1 + p_1\left[1 - p_2\,t\left(arctg(p_3 - p_2\,t) - \frac{\pi}{2}\right)\right] . \tag{8}$$

It bears the above-mentioned characteristic feature, i.e. tends to 1 at $t \to -\infty$. At large and positive t (8) mimicrizes the "stringy" behavior, i.e. grows linearly with t. We do not give a great significance to this, though. Moreover, such a behavior, if taken seriously, leads to complex singularities of $\alpha_P(t)$ and one has to provide a special care to save the amplitude from causality (analyticity) violations [35].



Residues are chosen as

$$\beta_P(t) = b_1 e^{b_2\, t}(1 - b_3\, t^3)\,, \quad \beta_f(t) = d_1 e^{d_2\, t}\,. \tag{9}$$

Our parametrization for the f_2-reggeon trajectory contains the QCD-inspired expression [24]

$$\alpha_f(t) = \left(\frac{8}{3\pi}\alpha_s(t - c_f)\right)^{1/2}\,, \tag{10}$$

where

$$\alpha_s(q^2) = \frac{4\pi}{11 - \frac{2}{3}n_f}\left(\frac{1}{\ln\frac{-q^2}{\Lambda^2}} + \frac{1}{1 + \frac{q^2}{\Lambda^2}}\right) \tag{11}$$

is the so-called one-loop analytical QCD effective coupling constant [36], $n_f = 3$ is the number of quark flavors taken into account, $\Lambda = 0.346\,GeV$ is the QCD dimensional parameter (the value was taken from [37]) and $c_f > 0$ is a free phenomenological parameter.

This analytic approximation for the f_2-reggeon trajectory is obtained in the following way. We take an expression for the $q\bar{q}$ Regge pole in the perturbative sector [24] derived via solving the Bethe-Salpeter-like equation in the range of the perturbative QCD validity and then replace the perturbative effective coupling constant to expression (11) derived in the framework of the dispersive approach [36]. Further, the introduction of the only free parameter $c_f > 0$ is the simplest phenomenological way to take into account the difference between the f_2-reggeon and a_2-reggeon intercepts and not to spoil the asymptotic behavior of the trajectory in the perturbative sector ($-t >> 1\,GeV^2$). We have to emphasize that approximation (10) is valid only at $t \le 0$ (in fact, the range of validity may be a little wider).

To obtain angular distributions we substitute (8), (9), (10) into (7), proceed via Fourier-Bessel transformation

$$f(b) = \frac{1}{16\pi s}\int_0^\infty d(-t) J_0(b\sqrt{-t}) f(t)\,, \quad f(t) = 4\pi s \int_0^\infty db^2 J_0(b\sqrt{-t}) f(b) \tag{12}$$

to the coordinate representation, using the eikonal representation of the scattering amplitude

$$T(s,b) = \frac{e^{2i\delta(s,b)} - 1}{2i} \tag{13}$$

through inverse Fourier-Bessel transformation (12) obtain its value in the momentum representation (during numerical calculating integrals from (12) we change upper limits of integration to $4\,GeV^2$ and $400\,GeV^{-2} \approx (4\,Fm)^2$ correspondingly) and substitute it into the expression for the differential cross-section

$$\frac{d\sigma}{dt} = \frac{|T(s,t)|^2}{16\pi s^2} \ . \tag{14}$$

Point out that the choice of parametrization (8), (9) is conditioned only by its simplicity and our desire to reproduce if approximately available experimental data on elastic scattering only in the range $\sqrt{s} > 50\,GeV$, $0.01\,GeV^2 < -t < 3\,GeV^2$. It has no fundamental physical content and may be replaced to another one providing more successful phenomenological description of the data on elastic scattering.

The fitting results

Turn to the description of experimental data. The results of fitting are represented in tab.1 and fig.1,2. In the fitting procedure there were used 523 experimental points taken from [38]. We have obtained $\chi^2/N_{points} = 4.99$.

	pomeron		**f-reggeon**
$p1$	0.144	c_f	0.227
$p2$	$1.64\,GeV^{-2}$		
$p3$	0.268		
$b1$	32.5	$d1$	327
$b2$	$2.56\,GeV^{-2}$	$d2$	$5.75\,GeV^{-2}$
$b3$	$0.194\,GeV^{-6}$		
$\alpha_P(0)$	1.144	$\alpha_f(0)$	0.728
$\alpha'_P(0)$	$0.309\,GeV^{-2}$	$\alpha'_f(0)$	$0.293\,GeV^{-2}$

Table 1: Parameters obtained by fitting to the data (9 adjustable parameters, $N_{points} = 523$, $\chi^2/N_{points} = 4.99$).

Point out that although the f_2-reggeon intercept value in our scheme ($\alpha_f(0) = 0.728$) is higher than that one from the rough linear approximation based on the appropriate resonance data (0.685) the corresponding slope ($\alpha'_f(0) = 0.29\,GeV^{-2}$) turns out noticeably smaller than in the linear scheme ($0.81\,GeV^{-2}$). So, our result does not imply any artificial behavior of the true f_2-reggeon trajectory at low positive values of the argument (see fig.4).

If we compare angular distributions obtained using parametrization (8), (9), (10) with those obtained using the same values of free parameters but with replacement of the nonlinear pomeron trajectory to its linear approximation (only two first terms in the Taylor expansion) we will disclose a huge difference between the corresponding results. It is also demonstrated in fig.1,2. Now we have some phenomenological ground to state that if we consider Regge trajectories nonlinear at negative values of the argument their nonlinearity should not be neglected when considering elastic diffraction.

In fig.3 the predictions for the total cross-section dependence on the center of mass energy are shown. In particular, $\sigma_{tot}(200\,GeV) \approx 50.2\,mb$, $\sigma_{tot}(14\,TeV) \approx 121\,mb$. Also, in this figure by comparison of the imaginary part of the forward amplitude with the same from the Born approximation it is demonstrated that we can not neglect absorptive corrections.

In the last figure the "soft" pomeron and f_2-reggeon trajectories as functions of the transferred momentum squared are presented.

We find it proper to exhibit the χ^2-quality of other authors who described the same processes as we do in this paper.

Ref.	$\chi^2/d.o.f.$	kinematical range
[4]	Not presented	$23\,GeV \leq \sqrt{s} \leq 546\,GeV$
[5]	2.04	$53\,GeV \leq \sqrt{s} \leq 630\,GeV\,, \quad 0 < -t \leq 5\,GeV^2$
[6]	2.38	$19.4\,GeV \leq \sqrt{s} \leq 630\,GeV\,, \quad 0.1\,GeV^2 \leq -t \leq 14.2\,GeV^2$
[7]	Not presented	$23\,GeV \leq \sqrt{s} \leq 546\,GeV$
[8]	4.27	$23\,GeV \leq \sqrt{s} \leq 630\,GeV\,, \quad 0 < -t \leq 6\,GeV^2$
[9]	Not presented	$546\,GeV \leq \sqrt{s} \leq 1800\,GeV$
[10]	2.83	$23.5\,GeV \leq \sqrt{s} \leq 1800\,GeV\,, \quad 0.01\,GeV^2 \leq -t \leq 14\,GeV^2$
[29]	2.60	$23.5\,GeV \leq \sqrt{s} \leq 1800\,GeV\,, \quad 0.01\,GeV^2 \leq -t \leq 14\,GeV^2$

Table 2: Some information on $\chi^2/d.o.f.$ over the data on elastic nucleon-nucleon scattering obtained by other authors.

Now we can discuss the correspondence of the Kwiecinski $q\bar{q}$-pole [24] to the f_2-reggeon trajectory and the relevance of the Kirschner-Lipatov "hard" pomeron pole [31] to elastic diffraction at accessible energies and small scattering angles. It is evident that the Kwiecinski pole (10) continued analytically in some way to the non-perturbative sector (a line with intercept lower than unity in fig.4) corresponds to the phenomenological f_2-reggeon trajectory giving a noticeable contribution to the observed elastic diffraction cross-sections. In other words, there exists a simple analytical way to connect the asymptotic perturbative behavior with the Regge phenomenology in the non-perturbative sector.

With the Kirschner-Lipatov "hard" pomeron the situation is quite different (see section "The QCD pomeron"). Trying to apply the Kirschner-Lipatov pole to phenomenology we have an alternative: either to insist on the correspondence between the "soft" pomeron trajectory and the Kirschner-Lipatov pole and so to accept that the trajectory is not monotonous in the Euclidean domain (in this case we come to the situation when we can not exploit its expression in form (5) for soft diffraction at any energies) or to assume that these poles are different ones (and so to presume that the Kirschner-Lipatov pole contribution to the eikonal is suppressed in the residue in the non-perturbative range of the argument value – it must dominate in the diffraction sector at ultra-high energies (higher than $1.8\,TeV$) and also in the perturbative sector at accessible energies). The latter variant is more preferable from the analytical point of view but in both cases we are not able to use (5) in the Regge phenomenology of diffraction phenomena at accessible energies. As to (6) it gives even less reasons to use it, as explained above. The following picture seems to us reasonable. At "very high" $(-t)$ we deal with pure gluon exchanges, then, with $(-t)$ diminishing, we come to "partially collectivized" exchanges in the form of "hard pomerons" (5). The latter could be seen in gluon-gluon elastic scattering with colorless exchanges ("Mueller-Navelet jets") with high $(-t)$. The hadron diffraction at $(-t) \leq 3\,GeV^2$ is dominated by "soft" or non-perturbative pomerons (reggeons) which cannot be thought as composed of definite number of partons. Unfortunately, QCD-literature has no much to say in this case.

Conclusions

So, in the framework of the Regge-eikonal model with using simple parametrizations for Regge trajectories and residues in which their asymptotic properties and fundamental nonlinearity are taken into account

1) it was shown that the diffractive pattern for the elastic $\bar{p}\,p$ and $p\,p$ scattering at energies $50\,GeV < \sqrt{s} < 2\,TeV$ and scattering angles $0.01\,GeV^2 < -t < 3\,GeV^2$ is mainly formed by contribution of the two C-even Regge trajectories,

2) it was demonstrated (from both theoretical and phenomenological points of view) that we can not ignore the nonlinearity of the trajectories in the considered kinematical range,

3) the relevance of the Kwiecinski $q\bar{q}$-pole to the f_2-reggeon trajectory and the impossibility to apply the Kirschner-Lipatov trajectory to elastic diffraction at accessible energies was argued.

At the end we would like to emphasize that obtained numerical results are preliminary and can be modified both by selection of new parametrizations for the poles and residues and via adding other reggeon contributions to the eikonal.

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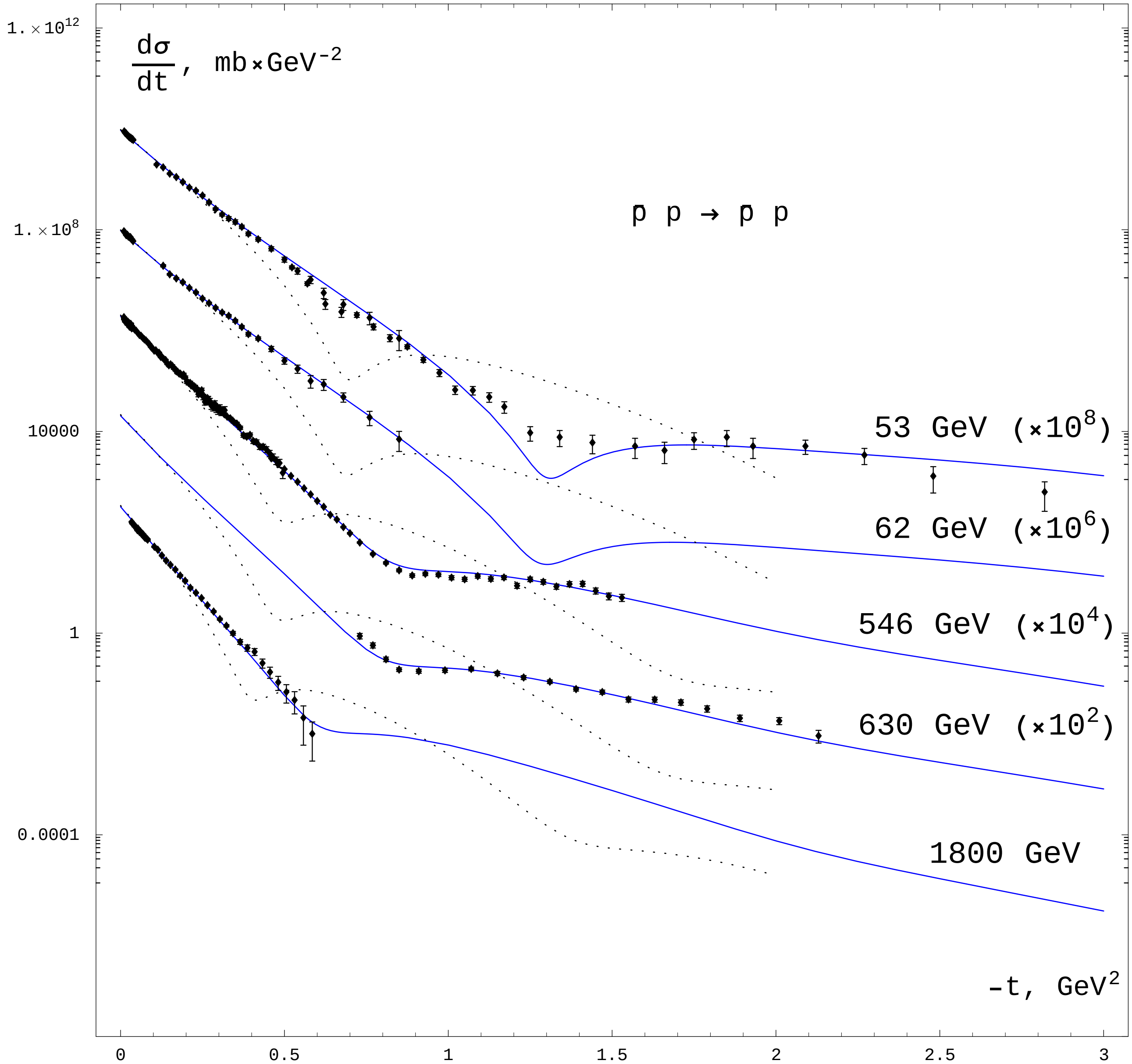

Figure 1: Differential $\bar{p}\,p \to \bar{p}\,p$ cross-sections for the cases of nonlinear (solid lines) and linearly approximated (dotted lines) "soft" pomeron trajectory.

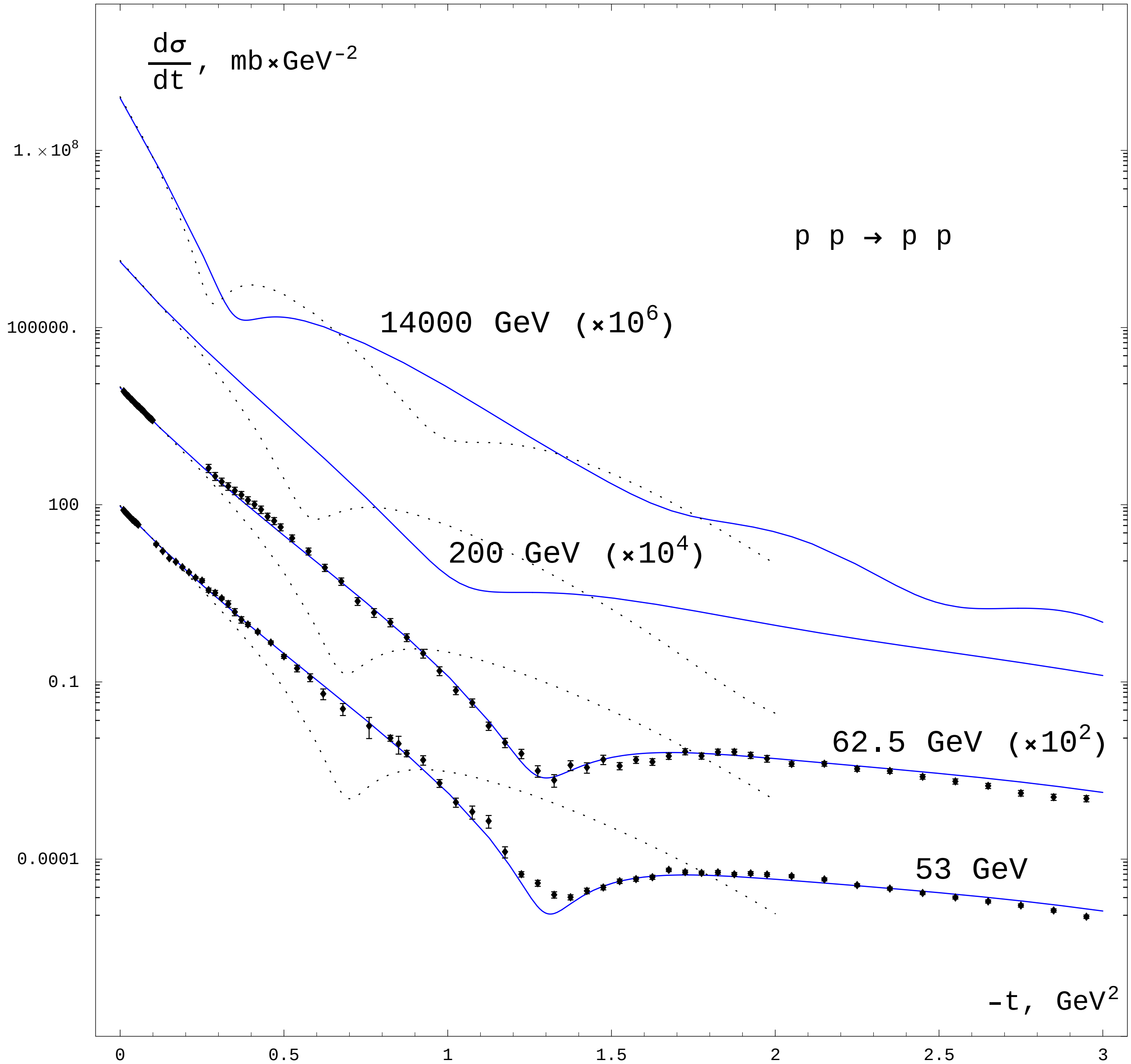

Figure 2: Differential $p\,p \to p\,p$ cross-sections for the cases of nonlinear (solid lines) and linearly approximated (dotted lines) "soft" pomeron trajectory.

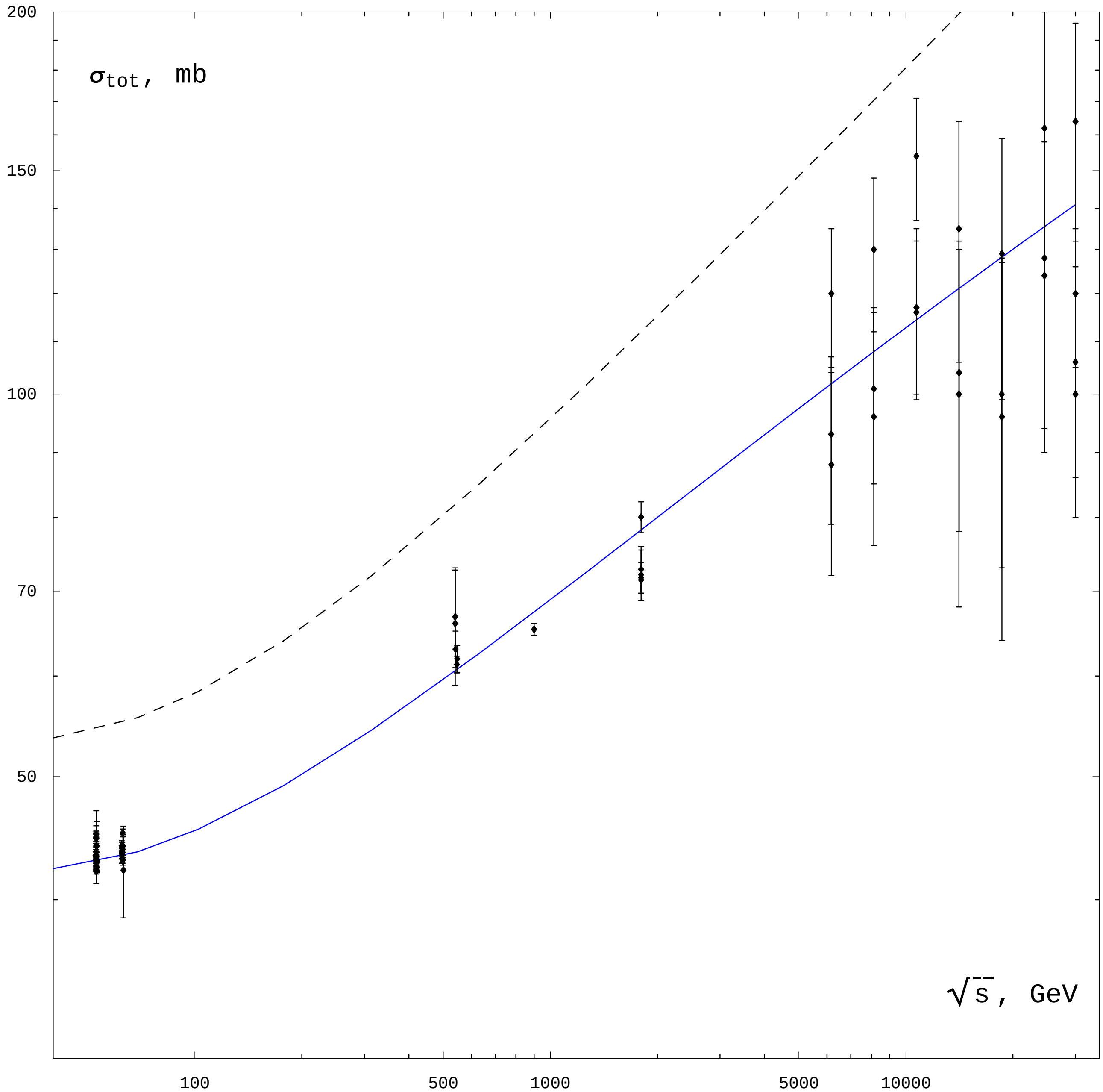

Figure 3: Total cross-section (the eikonalized amplitude — solid line, the Born amplitude — dashed line) for high-energy nucleon-nucleon scattering as a function of center-of-mass energy (experimental data were taken from Particle Physics Data System http://wwwppds.ihep.su:8001/ppds.html).

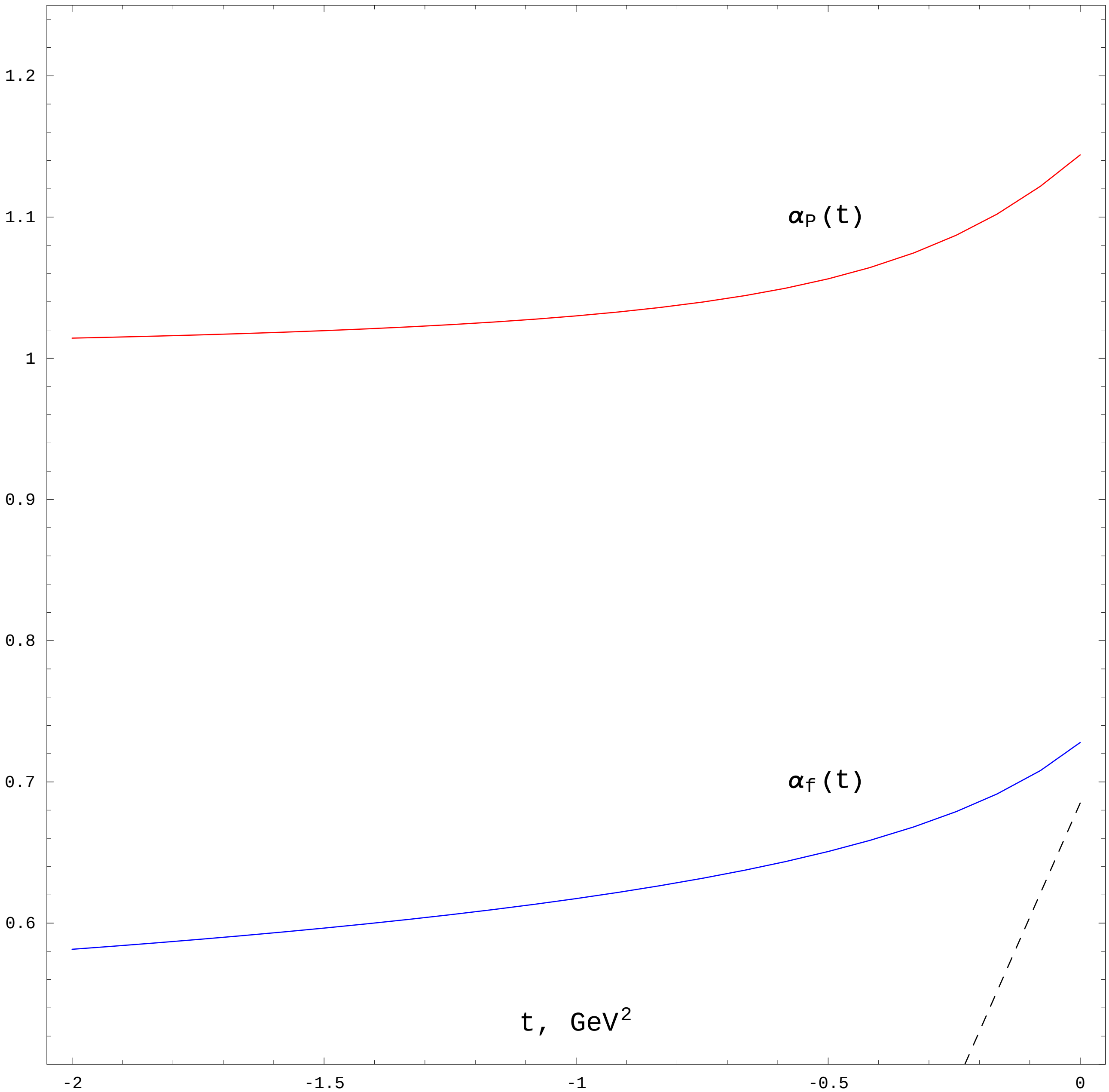

Figure 4: "Soft" pomeron and f_2-reggeon trajectories obtained in the fitting procedure (dashed line — $\alpha_f^{lin}(t) = 0.685 + 0.8084\,t$ is the continuation of the Chew-Frautschi plot corresponding to f_2-reggeon).